UNIVERSITY OF GOUR BANGA  
P.O.- Mokdumpur, Malda 732103, West Bengal

Syllabus for Research Eligibility Test (RET)  
in Mathematics-2018

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Detailed Syllabus for RET in Mathematics:

**GROUP A: RESEARCH METHODOLOGY:**

General Research Methodology: Ethical values of research, ideology of research, tools and techniques of research, writing of a research proposal/ project and paper, writing of references of research paper, plagiarism of a research paper. Presentation of a research paper (oral, poster etc.), fast track communication, short communication, letter to the editor of a research paper in a journal, research article, review article, review of a research paper, citation of a research paper.

Subject Oriented Methodology: Axioms, postulates, convention, hypothesis, statement, logic, proposition, theorem, methods of proof of a statement, characterization and classification of a concept, algorithm, unsolved problems, open problems. Goedel incompleteness theorem, postulates of Euclidean geometry, historical development of non- Euclidean geometries. Subject indexing, Mathematics Subject Classification and Mathematical Reviews of American Mathematical Society, ISSN of journals, peer review articles, impact factor of a journal, article.
GROUP B: MATHEMATICS:


Rings: Ideals and Homomorphisms, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Polynomial and Power Series Rings. Divisibility Theory: Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss’ Theorem, Eisenstein’s criterion.


Real Analysis: Bounded Variation: Functions of Bounded Variation and their properties, Riemann Stieltjes integrals and its properties, Absolutely Continuous Functions.

The Lebesgue Measure: Lebesgue Measure: (Lebesgue) Outer measure and measure on $\mathbb{R}$, Measurable sets form an $\sigma$-algebra, Borel sets, Borel $\sigma$-algebra, open sets, closed sets are measurable. Existence of a non-measurable set, Measure space, Measurable Function and its properties, Borel measurable functions, Concept of Almost Everywhere (a.e.), sets of measure zero, Steinhaus Theorem, Sequence of measurable functions, Egorov’s Theorem, Applications of Lusin Theorem.

Ordinary Differential Equations & Special Functions: Ordinary Differential Equations: Existence and Uniqueness: First order ODE, Initial value problems, Existence theorem, Uniqueness, basic theorems, Ascoli Arzela theorem (statement only), Theorem on convergence of solution of initial value problems, Picard-Lindelöf theorem (statement only), Peano’s existence theorem (statement only) and corollaries.

Higher Order Linear ODE: Higher order linear ODE, fundamental solutions, Wronskian, variation of parameters.


Hypergeometric Equation: Hypergeometric Functions, Series Solution near zero, one, and infinity. Integral Formula, Differentiation of Hypergeometric Function.


Bessel Equation: Bessel’s Functions, Series Solution, Generating Function, Integral Representation of Bessel’s Functions, Hankel Functions, Recurrence Relations, Asymptotic Expansion of Bessel Functions.

Calculus of Several Variables: $\mathbb{R}^n$ as a normed linear space, and $L(\mathbb{R}^n, \mathbb{R}^m)$ as a normed linear space. Limits and continuity of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$. The derivative at a point of a function from $\mathbb{R}^n$ to $\mathbb{R}^m$ as a linear transformation. The tangent space and linear approximation. The chain rule. Partial derivatives and higher order partial derivatives and their continuity. Sufficient conditions for differentiability. Comparison between the differentiability of a function from $\mathbb{R}^2$ to $\mathbb{R}^2$ and from $C$ to $C$. Examples of discontinuous and non-differentiable functions whose partial derivatives exist. $C^1$ maps. Euler’s theorem. Sufficient condition for equality of mixed partial derivatives. Proofs of the Inverse Function Theorem, the Implicit Function Theorem, and the Rank Theorem. Jacobians. The Hessian and the real quadratic form associated with it. Extrema of real-valued functions of several variables. Proof of the necessity of the Lagrange multiplier condition for constrained extrema. Riemann Integral of real-valued functions on Euclidean spaces, measure zero sets, Fubini’s Theorem. Partition of unity, change of variables. Stokes’ Theorem and Divergence Theorem for integrals.

Complex Analysis: Complex Numbers: Complex Plane, Stereographic Projection.

Complex Differentiation: Derivative of a complex function, Comparison between differentiability in the real and complex senses, Comparison between the real and complex differentiability via $\mathbb{R}$-linear and $\mathbb{C}$-linear maps, Cauchy-Riemann equations, Necessary and sufficient criterion for complex differentiability, Analytic functions, Entire functions, Harmonic functions and Harmonic conjugates.

Complex Functions and Conformality: Polynomial functions, Rational functions, Power series, Exponential, Logarithmic, Trigonometric and Hyperbolic functions, Branch of a logarithm, Conformal maps, Mobius Transformations.
Complex Integration: The complex integral (over piecewise $C^1$ curves), Cauchy’s Theorem and Integral Formula, Power series representation of analytic functions. The difference between Real Analytic functions and $C^n$-functions over $R$. Real Analyticity vs. Complex Analyticity. Morera’s Theorem, Goursat’s Theorem, Liouville’s Theorem, Fundamental Theorem of Algebra, Zeros of analytic functions, Identity Theorem, Weierstrass Convergence Theorem, Maximum Modulus Principle and its applications, Schwarz’s Lemma, Index of a closed curve, Contour, Index of a contour, Simply connected domains, Cauchy’s Theorem for simply connected domains.

Singularities: Definitions and Classification of singularities of complex functions, Isolated singularities, Uniform convergence of sequences and series. Laurent series, Casorati-Weierstrass Theorem, Poles, Residues, Residue Theorem and its applications to contour integrals, Meromorphic functions, Applications of Argument Principle, Applications of Rouche’s Theorem.


(Infinitite) Product Topology: Sub basis for product Topology defined by Projection Maps, Box Topology, Metric Topology.

Connectedness and Compactness: Connected and Path Connected Spaces: Definitions, Examples and its simple properties, Connected subsets of the real line, Introduction to Components and Path Components, Local Connectedness.

Compact Spaces, Compact subsets of the real line, Heine-Borel Theorem.

Second Order Linear P.D.E.: Classification, reduction to normal form; Solution of equations with constant coefficients by (i) factorization of operators (ii) separation of variables.
Parabolic Differential Equations: Formation and solution of Diffusion equation, Dirac, Delta function, Separation of variables method, Solution of Diffusion Equation in Cylindrical and spherical coordinates, Examples.

Hyperbolic Differential Equations: Formation and solution of one-dimensional wave equation, canonical reduction, Initial Value Problem, D’Alembert’s solution, Vibrating string, Forced Vibration, Initial Value Problem and Boundary Value Problem for two-dimensional wave equation, Periodic solution of one-dimensional wave equation in cylindrical and spherical
coordinate systems, vibration of circular membrane, Uniqueness of the solution for the wave equation, Duhamel’s Principle, Examples.

Green’s Function: Green’s function for Laplace Equation, methods of Images, Eigen function Method, Green’s function for the wave and Diffusion equations. Laplace Transform method: Solution of Diffusion and Wave equation by Laplace Transform.

**Functional Analysis:** Banach Spaces: Normed Linear Spaces and its properties, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Riesz Lemma. Bounded Linear Transformations. Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Necessary and sufficient conditions for Bounded (Continuous) and Unbounded (Discontinuous) Linear functionals in terms of their kernel. Hyperplane, Necessary and sufficient conditions for a subspace to be hyperplane. Applications of Hahn-Banach Theorem, Dual Space, Examples of Reflexive Banach Spaces. $L^p$-Spaces and their properties.


**Numerical Analysis:** Numerical Solution of System of Linear Equations: Triangular factorization methods, Matrix inversion method, Iterative methods- Jacobi method, Gauss Jacobi method, Gauss-Seidel method, Successive over relaxation (SOR) method and convergence condition of Iterative methods, Rate of convergence of methods.

Solution of Non-linear Equations: Iteration methods: Tchebyshev method, Multipoint method, Modified Newton-Raphson method (for real roots simple or repeated), Rate of convergence of all iteration methods.


Two Point Boundary Value Problem for ODE: Finite difference method, Shooting Method.
Numerical Solution of PDE by Finite Difference Method: Parabolic equation in one dimension (Heat equation), Explicit finite difference method, Implicit Crank Nickolson method, Hyperbolic equation in one-space dimension (Wave equation)- Finite difference method, Convergence and Stability.

**Mathematical Methods:** Laplace Transform: Laplace transform, properties of Laplace transform, inversion formula of Laplace transform (Bromwich formula), Convolution theorem, Application to ordinary and partial differential equations.

Fourier Transform: Properties of Fourier transform, Inversion formula, Convolution, Parseval’s relation, Multiple Fourier transform, Bessel’s inequality, Application of transform to Heat, Wave and Laplace equations.


Integral Equation: Basic concepts, Volterra integral equations, Relationship between linear differential equations and Volterra equations, Resolvent kernel, Method of successive approximations, Convolution type equations, Volterra equation of first kind, Abel’s integral equation, Fredholm integral equations, Fredholm equations of the second kind, the method of Fredholm determinants, Iterated kernels, Integral equations with degenerate kernels, Eigen values and Eigen functions of a Fredholm alternative, Construction of Green’s function for Boundary Value Problems, Singular integral equations.